

COURSE TITLE

PreCalculus

LENGTH

Full Year

DEPARTMENT

STEM Department

SCHOOL

Rutherford High School

DATE

Fall 2018

PreCalculus

I. Introduction/Overview/Philosophy

Students in PreCalculus will study concepts in trigonometry and advanced algebra in preparation for calculus at the college level. Both algebraic and transcendental functions will be reviewed, investigated and applied at a variety of levels: numerically, graphically, and analytically. Students will regularly apply the tools of technology including the graphing calculator and computer to solve problems. They will be challenged through critical thinking exercises and participate in various group and individual activities that will enhance their mathematical reasoning ability and communication skills.

II. Objectives

Course Outline:

1. Trigonometry
 - a. Radian/Degrees, complements/supplements, co terminal angles
 - b. Unit Circle
 - c. Right triangle Trig
 - d. Graphing Trig Functions
 - e. Applications
2. Analytic Trigonometry
 - a. Using and verifying Fundamental Identities
 - b. Solving Trig Equations
 - c. Multiple angle problems
3. Additional Topics in Trigonometry
 - a. Law of Sines
 - b. Law of Cosines
 - c. Application of both in real life contact
 - d. Vectors
4. Functions and their Graphs
 - a. Linear Equations and functions
 - b. Analyzing Graphs of functions and their transformations
 - c. Combination of Functions
 - d. Inverse Functions
5. Polynomial and Rational Functions
 - a. Quadratic Functions and Models
 - b. Polynomial Functions of a Higher Degree
 - c. Polynomial Synthetic Division
 - d. Complex Numbers
 - e. Zeros of a Polynomials
 - f. Rational Functions
6. Exponential and Logarithmic Functions and their Graphs
 - a. Domain and Range for these graphs
 - b. Applications for growth and Decay models

- c. Properties of Logarithms
 - d. Exponential and Logarithmic Equations
7. Conic Sections
- a. Distance between a point and a line
 - b. Parabolas
 - c. Ellipses
 - d. Hyperbolas
8. Introduction to limits
- a. Sequences and Series
 - b. Finding Limits: charts graphically and algebraically
 - c. Limits at Infinity
 - d. Difference Quotient

Student Outcomes:

After successfully completing this course, the student will:

- Understand and apply the concept of the circular functions.
- Gain an appreciation for the graphs of the trigonometric functions and their interrelation.
- Understand the relationship between the modern and traditional approach to trigonometry.
- Be able to solve trigonometric problems and to graph trigonometric functions, and to use the calculator appropriately.
- Understand the relationship of transformations to functions and vectors.
- Review, reinforce, and investigate algebraic, exponential, and logarithmic functions and the concepts learned in algebra and geometry in order to prepare for the study of calculus.

New Jersey Student Learning Standards

CAREER READY PRACTICES

CRP1 Act as a responsible and contributing citizen and employee.

Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

CRP2 Apply appropriate academic and technical skills.

Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

CRP4 Communicate clearly and effectively and with reason.

Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting

with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

CRP6. Demonstrate creativity and innovation.

Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP7. Employ valid and reliable research strategies.

Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP11. Use technology to enhance productivity.

Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

TECHNOLOGY

Standard 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Strand F: Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

8.1.12.F.1- Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

Standard 8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

Strand E. Computational Thinking: Programming: Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

8.2.12.E.1- Demonstrate an understanding of the problem-solving capacity of computers in our world.

21ST CENTURY LIFE AND CAREERS**9.1 Personal Financial Literacy****Strand B: Money Management**

9.1.12.B.1 Prioritize financial decisions by systematically considering alternatives and possible consequences.

9.1.12.B.2 Compare strategies for saving and investing and the factors that influence how much should be saved or invested to meet financial goals.

9.1.12.B.8 Describe and calculate interest and fees that are applied to various forms of spending, debt, and saving.

9.2 Career Awareness, Exploration, and Preparation**Strand C: Career Preparation**

9.2.12.C.1 Review career goals and determine steps necessary for attainment.

9.2.12.C.3 Identify transferable career skills and design alternate career plans.

NEW JERSEY STUDENT LEARNING STANDARDS- MATH

A-APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

A-REI.D.12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A-REI.B.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

A-SSE-A.1. Interpret expressions that represent a quantity in terms of its context.1

A-SSE-A.2. Use the structure of an expression to identify ways to rewrite it.

A-SSE-B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

F-BF.A.1. Write a function that describes a relationship between two quantities.

F-BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4. Find inverse functions.

F-BF.B.5. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.C.7d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F-IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-LE.A.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

F-TF.A.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.

F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.C.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

F-TF.C.9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G-GPE.A.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.A.2. Derive the equation of a parabola given a focus and directrix.

G-GPE.A.3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

G-SRT.C.7. Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ➡

G-SRT.D.10. Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.D.9. Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

N-CN.A.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN.A.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N-Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

III. Proficiency Levels

PreCalculus is available to students who have successfully completed Algebra 2 or its equivalent.

IV. Methods of Assessment

Student Assessment

The teacher will provide a variety of assessments during the course of the year. The assessment may include but is not limited to: chapter and unit tests and quizzes, teacher observations, open-ended problems, cooperative work, and homework.

Curriculum/Teacher Assessment

The teacher will provide the subject area supervisor with suggestions for changes on an ongoing basis.

V. Grouping

PreCalculus is a heterogeneously grouped junior/senior level course.

VI. Articulation/Scope & Sequence/Time Frame

Course length is one year.

VII. Resources

Texts/Supplemental Reading/References

PreCalculus, 5th Edition, Houghton Mifflin Company, 2001

VIII. Suggested Activities

Appropriate activities are listed in the curriculum map.

IX. Methodologies

The following methods of instruction are suggested: teacher guided explorations, working in groups/working with a partner, working with manipulatives and discovery activities.

X. Interdisciplinary Connections

Connections are made to science, particularly physics, by means of projects coordinating topics in the two subject areas. Connections are also made by means of formulas used in computer programming classes. Writing assignments and portfolios strengthen the connection between mathematics and language arts literacy and fine arts.

XI. Differentiating Instruction for Students with Special Needs: Students with Disabilities, Students at Risk, English Language Learners, and Gifted & Talented Students

Differentiating instruction is a flexible process that includes the planning and design of instruction, how that instruction is delivered, and how student progress is measured. Teachers recognize that students can learn in multiple ways as they celebrate students' prior knowledge. By providing appropriately challenging learning, teachers can maximize success for all students.

Differentiating in this course includes but is not limited to:

Differentiation for Support (ELL, Special Education, Students at Risk)

- Peer mentoring on problems
- Differentiated teacher feedback on assignments
- Modeling out problems on whiteboard
- Visual aids as we project problems on whiteboard
- Study guides
- Tiered assignments
- Scaffolding of materials and assignments
- Re-teaching and review

- Guided note taking
- Exemplars of varied performance levels
- Multi-media approach to accommodating various learning styles

Differentiation for Enrichment

- Supplemental reading material for independent study
- Flexible grouping
- Tiered assignments
- Topic selection by interest
- Enhanced expectations for independent study
- Elevated questioning techniques using Webb's Depth of Knowledge matrix

XII. Professional Development

The teacher will continue to improve expertise through participation in a variety of professional development opportunities.

XII. Curriculum Map/Pacing Guide

| Unit Topic | Time Allocated | Differentiating Instruction for Students with Disabilities, Students at Risk, English Language Learners, & Gifted & Talented Students | Standards | Assessments |
|---|----------------|---|---|---|
| Trigonometry <ul style="list-style-type: none"> • Radian/Degrees, complements/supplements, coterminal angles • Unit Circle • Right triangle Trig • Graphing Trig Functions • Applications | 8 weeks | <i>For Support:</i> Khan Academy, IXL, Class notes with examples, pairing <i>For Enhancement:</i> Khan Academy, IXL Real world problems, | G.SRT.7, G.SRT.8, F.TF.1, F.TF.2, F.TF.3, F.TF.8 F.IF7 CRP1,2,4,6,7,8,11 8.1.12.F.1 8.2.12.E.1 | <i>Formative Assessment:</i> Projects, class discussions, presentations, student centered activities <i>Summative Assessment</i> Tests-Unit Circle and finding angles, quizzes, Labs graphing the six trig functions |
| Analytic Trigonometry <ul style="list-style-type: none"> • Using and verifying Fundamental Identities • Solving Trig Equations • Multiple angle problems | 5 weeks | <i>For Support:</i> Khan Academy, IXL, teacher modeling, class notes with examples <i>For Enhancement:</i> More difficult problems, Khan Academy, independent work | F.TF.3, F.TF.8, F.TF.5, F.TF.8, F.TF.9 CRP1,2,4,6,7,8,11 | <i>Formative Assessment:</i> Homework, classwork, questioning, group work, class discussion <i>Summative Assessment</i> Test Verifying Identities, Quizzes, simplifying identities, solving trig equations Benchmark Q1 |
| Additional Topics in Trig <ul style="list-style-type: none"> • Law of Sines • Law of Cosines • Application of both in | 4 weeks | <i>For Support:</i> Khan Academy, IXL, Technology, Teacher modeling | SRT.D.9, SRT.D.10 CRP1,2,4,6,7,8,11 | <i>Formative Assessment</i> Homework, classwork, cooperative group work <i>Summative Assessment</i> |

| Unit Topic | Time Allocated | Differentiating Instruction for Students with Disabilities, Students at Risk, English Language Learners, & Gifted & Talented Students | Standards | Assessments |
|---|----------------|--|--|--|
| real life contact • Vectors | | <i>For Enhancement:</i> Khan Academy, independent work, real life applications | | Test Law of Sines and Cosine, Quiz Application of law of sine and cosine Quiz Vectors |
| Functions and their Graphs • Linear Equations and functions • Analyzing Graphs of functions and their transformations • Combination of Functions • Inverse Functions | 4 weeks | <i>For Support:</i> IXL, cooperative group work, technology, video pod casts <i>For Enhancement:</i> Khan Academy, student driven projects, independent practice, more difficult problems | A.APR.1, F.FI.7, F.BF.1, F.BF.4, A.CED.2, CRP1,2,4,6,7,8,11 | <i>Formative Assessment:</i> Homework, classwork, group activities, IXL, Discussions on graphs <i>Summative Assessment</i> Test Chapter 1 Functions, quizzes: graphing functions, quiz inverses Benchmark Q2 |
| Polynomial and Rational Functions • Quadratic Functions and Models • Polynomial Functions of a Higher Degree • Polynomial Synthetic Division • Complex Numbers • Zeros of a Polynomials • Rational Functions | 4 weeks | <i>For Support:</i> Teacher modeling, pair grouping, IXL, note package with examples <i>For Enhancement:</i> More difficult problems, Individual project, Khan Academy, Real world problems | A.REI.4a, A.REI.7, F.IF.4, F.IF.5, F.IF.7 A.APR.1, A.APR.2, N.CN.2, N.CN.3, CRP1,2,4,6,7,8,11 8.1.12.F.1 8.2.12.E.1 | <i>Formative Assessment:</i> Homework, classwork, IXL, project <i>Summative Assessment</i> Test Polynomials, Test Rational Function Quiz graphing polynomials Quiz graphing rational functions |
| Exponential and Logarithmic Functions | 5 weeks | <i>For Support:</i> IXL, group work, authentic | F.IF7d, F.IF.7e, | <i>Formative Assessment:</i> Homework, class work, exit |

| Unit Topic | Time Allocated | Differentiating Instruction for Students with Disabilities, Students at Risk, English Language Learners, & Gifted & Talented Students | Standards | Assessments |
|--|----------------|---|---|--|
| and their Graphs <ul style="list-style-type: none"> • Domain and Range for these graphs • Applications for growth and Decay models • Properties of Logarithms • Exponential and Logarithmic Equations | | assessment, teacher modeling <i>For Enhancement:</i> Individual work, Khan Academy, real world application problems | F.BF.5, F.LE.4, A.REI.10, A.REI.11, A.REI.12 CRP1,2,4,6,7,8,11 8.1.12.F.1 8.2.12.E.1 9.1.12.B.1 9.1.12.B.2 9.1.12.B.8 | questions, IXL <i>Summative Assessment</i> Quiz graphing logs and exponential functions Test Property of Logs and their applications Benchmark Q3 |
| Conic Sections <ul style="list-style-type: none"> • Distance between a point and a line • Parabolas • Ellipses • Hyperbolas | 5 weeks | <i>For Support:</i> IXL, group work, teacher modeling, posted notes <i>For Enhancement:</i> More difficult problems, individual packets, real world problems | GPE.A.1, GPE.A.2, GPE.A.3 CRP1,2,4,6,7,8,11 | <i>Formative Assessment</i> Homework, classwork, teacher questioning, Khan Academy <i>Summative Assessment</i> Quiz Distance Quiz Parabolas and Circles Quiz Ellipse and Hyperbola Test Conics |
| Introduction to limits <ul style="list-style-type: none"> • Sequences and Series • Finding Limits: charts graphically and algebraically • Limits at Infinity • Difference Quotient | 5 weeks | <i>For Support:</i> IXL, cooperative group work, pairing, Labs <i>For Enhancement:</i> Real world applications, interest based project, higher order thinking skills | N-Q.2-3, A-SSE.2-3, A-CED.3, F-IF.5 CRP1,2,4,6,7,8,11 9.2.12.C.1 9.2.12.C.3 | <i>Formative Assessment</i> Homework, class work, IXL, discussion <i>Formative Assessment</i> Test Limits Benchmark Q4 |