COURSE TITLE

Honors Calculus

LENGTH

Full Year

DEPARTMENT

STEM Department

SCHOOL

Rutherford High School

DATE

September 10, 2018

Initial BOE Approval Date (Born on): 6/15/2015

Honors Calculus

I. Introduction/Overview/Philosophy

Fourth-year students in the Honors program will understand the concepts of differential and integral calculus and be able to apply them in various problem-solving situations. The course provides a multi-representational approach to calculus, with concepts, results, and problems being expressed geometrically, numerically, analytically, and verbally. The connections among these methods of representation are also emphasized.

Students will regularly apply the tools of technology including the graphing calculator and computer to solve problems. They will be challenged through critical thinking exercises and participate in various group and individual activities that will enhance their mathematical reasoning ability and communication skills.

II. Objectives

Course Outline:

1. Acquire an intuitive understanding of the limiting process by:
    i. Calculating limits using algebra.
    ii. Estimating limits from graphs or tables of data.
    iii. Using the e-∂ definition of a limit and finding ∂ algebraically.
2. Analyze functions and their graphs by:
    i. Observing asymptotic behavior.
    ii. Calculating rates of change.
    iii. Determining continuity of functions.
3. Investigate and calculate derivatives by:
    i. Using the limit definition of a derivative.
    ii. Employing various techniques for differentiation.
    iii. Relating differentiability and continuity of functions.
    iv. Calculating rate of change and slope of tangent at a point.
    v. Comparing average rate of change to instantaneous rate of change.
4. Use the derivatives of a function by:
    i. Analyzing the characteristics of graphs of f and f'.
    ii. Studying the relationship between the increasing and decreasing behavior of f and the sign of f'.
    iii. Applying the Mean Value Theorem for derivatives.
    iv. Examining the interpretation of Rolle's Theorem.
    v. Solving involving first and second derivatives.
    vi. Relating concavity and the graphs of f, f', and f".
    vii. Employing the use of the normal line when examining concavity of a graph.
    viii. Finding points of inflection.
5. Apply derivatives by:
    i. Analyzing graphs of various functions.
    ii. Relating optimization with absolute and relative extrema.

iii. Solving problems involving related rates
iv. Using implicit differentiation to find the derivative of an inverse function.
v. Using the derivative as a rate of change in application areas, including velocity and acceleration.

6. Utilize techniques of antidifferentiation by:
   i. Recognizing the relationship between derivatives and integrals.
   ii. Using initial conditions in the solution of a differential equation.
   iii. Employing powers of odd and even trigonometric functions.
   iv. Using various substitution methods.
7. Investigate and calculate integrals by:
   i. Evaluating Riemann sums using left, right and midpoint evaluation points.
   ii. Calculating the definite integral as a limit of Riemann sums over equal subdivisions
   iii. Examining the definite integral as the change of quantity over the interval.
   iv. Applying basic properties of definite integrals.
8. Use the Fundamental Theorem of Calculus in the process of:
   i. Evaluating definite integrals
   ii. Representing a particular antiderivative and its analytical and graphical analyses.
9. Apply definite integrals to:
   i. Calculating the area of a region.
   ii. Estimating area by Riemann sum and Trapezoidal Rule.
   iii. Determining the distance traveled by a particle.
   iv. Calculating volume of a solid of revolution and by cross-section.
   v. Finding the average value of a function.
10. Explore transcendental functions by:
   i. Finding derivatives of inverses of functions.
   ii. Taking derivatives and integrals of exponential and logarithmic functions, specifically functions of the form e^u, a^u, ln u, and log u.
   iii. Using logarithmic differentiation.
   iv. Solving separable differential equations and using them in modeling real world phenomena, especially for logarithmic functions and exponential growth.

Student Outcomes:

After successfully completing this course, the student will:

- Work with functions represented in a variety of ways: graphically, numerically, analytically, or verbally.
- Understand the meaning and application of limits.
- Understand the relationship between continuity and differentiability.
- Understand the meaning of the derivative in terms of a rate of change and local linear approximation.
- Understand the meaning of the definite integral both as the limit of Riemann sums and as the net accumulation of a rate of change.
- Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Use derivatives to solve a variety of problems.
- Use integrals to solve a variety of problems.
- Communicate mathematics both orally and in well-written sentences and explain solutions to problems.
- Model a written description of a physical situation with a function, a differential equation, or an integral.
- Use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Determine the reasonableness of solutions.

- Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

New Jersey Student Learning Standards

CAREER READY PRACTICES

CRP1 Act as a responsible and contributing citizen and employee.

Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

CRP2 Apply appropriate academic and technical skills.

Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

CRP4 Communicate clearly and effectively and with reason.

Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

CRP6. Demonstrate creativity and innovation.

Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP7. Employ valid and reliable research strategies.

Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP11. Use technology to enhance productivity.
Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

TECHNOLOGY

Standard 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Strand A. Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
8.1.12.A.1- Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.

Standard 8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming: All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
Strand A. The Nature of Technology: Creativity and Innovation Technology systems impact every aspect of the world in which we live.
8.2.12.A.2- Analyze a current technology and the resources used, to identify the trade-offs in terms of availability, cost, desirability and waste.
Strand C. Design: The design process is a systematic approach to solving problems.
8.2.12.C.3- Analyze a product or system for factors such as safety, reliability, economic considerations, quality control, environmental concerns, manufacturability, maintenance and repair, and human factors engineering (ergonomics).
8.2.12.C.4- Explain and identify interdependent systems and their functions.
Strand D. Abilities for a Technological World: The designed world is the product of a design process that provides the means to convert resources into products and systems.
8.2.12.D.4- Assess the impacts of emerging technologies on developing countries.

21ST CENTURY LIFE AND CAREERS

9.1 Personal Financial Literacy
Strand A: Income and Careers
9.1.12.A.4 Identify a career goal and develop a plan and timetable for achieving it, including educational/training requirements, costs, and possible debt.
Strand E: Becoming a Critical Consumer
9.1.12.E.5 Evaluate business practices and their impact on individuals, families, and societies.
9.2 Career Awareness, Exploration, and Preparation
Strand C: Career Preparation
9.2.12.C.1 Review career goals and determine steps necessary for attainment.

NEW JERSEY STUDENT LEARNING STANDARDS- MATH

N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.C.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F-IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
A-SSE-B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A-SSE-B.4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A-APR.C.4. Prove polynomial identities and use them to describe numerical relationships.
A-REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A-REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.B.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A-REI.B.4. Solve quadratic equations in one variable.
F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
G-GMD.A.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
G-MG.A1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
G-MG.A.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

III. Proficiency Levels

Honors Calculus is available to those students who meet the stated criteria for placement.

IV. Methods of Assessment

Student Assessment
The teacher will provide a variety of assessments during the course of the year. The assessment may include but is not limited to: chapter and unit tests and quizzes, application problems, homework, hands on activities, and group work.

Curriculum/Teacher Assessment
The teacher will provide the subject area supervisor with suggestions for changes on an ongoing basis.

V. Grouping

Honors Calculus is a homogeneously grouped course at the twelfth-grade level.

VI. Articulation/Scope & Sequence/Time Frame

Course length is one year.

VII. Resources

Texts/Supplemental Reading/References

Resources include but are not limited to:

A. Text:
   1. Thomas, George B. and Finney, Ross L. Calculus and Analytic Geometry - 9th Edition. Reading, Massachusetts: Addison-Wesley Publishing Company, 1996.

B. References
   1. Ayres Schaum's Outline Series, Theory and Problems of Differential and Integral Calculus, 2nd edition McGraw-Hill Book Company, 1964.
   2. Forester Calculus, Concepts and Applications. Key Curriculum Press, 1998
   3. Varberg, Dale and Purcell, Edwin. Calculus with Analytic Geometry. Englewood Cliffs, New Jersey: Prentice Hall, 1992.

C. Software
   1. TI-89 Graphing Calculator
   2. Various related Internet websites.

VIII. Suggested Activities

Appropriate activities are listed in the curriculum map below.

IX. Methodologies

The following methods of instruction are suggested: lecture, working in groups/working with a partner, and discovery activities.

X. Interdisciplinary Connections

Connections are made to science, particularly physics, by means of collaborative projects coordinating topics in the two subject areas. Writing assignments and portfolios strengthen the connection between mathematics and language arts literacy and fine arts.

XI. Differentiating Instruction for Students with Special Needs: Students with Disabilities, Students at Risk, English Language Learners, and Gifted & Talented Students

Differentiating instruction is a flexible process that includes the planning and design of instruction, how that instruction is delivered, and how student progress is measured. Teachers recognize that students can learn in multiple ways as they celebrate students' prior knowledge. By providing appropriately challenging learning, teachers can maximize success for all students.

Differentiating in this course includes but is not limited to:

Differentiation for Support (ELL, Special Education, Students at Risk)

- Peer mentoring on problems
- Differentiated teacher feedback on assignments
- Modeling out problems on whiteboard
- Visual aids as we project problems on whiteboard
- Study guides
- Tiered assignments

- Scaffolding of materials and assignments
- Re-teaching and review
- Guided note taking
- Exemplars of varied performance levels
- Multi-media approach to accommodating various learning styles

Differentiation for Enrichment

- Supplemental reading material for independent study
- Flexible grouping
- Tiered assignments
- Topic selection by interest
- Enhanced expectations for independent study
- Elevated questioning techniques using Webb's Depth of Knowledge matrix

XII. Professional Development

The teacher will continue to improve expertise through participation in a variety of professional development opportunities.

XII. Curriculum Map/Pacing Guide

Unit Topic	Time Allocated	Differentiating Instruction for Students with Disabilities, Students at Risk, English Language Learners, & Gifted & Talented Students	Standards	Assessments
Limits ● Properties of Limits ● Finding Limits Graphically and Numerically ● Evaluating Limits Analytically ● Continuity and One Sided Limits ● Infinite Limits	4 weeks	*For Support:* Guided Notes, Use of IXL, Use of a Calculator, Modified Assessments Rephrase questions, directions, and explanations, Testing accommodations, *For Enhancement:* Use of IXL, Provide extension activities, Adjusting the pace of lessons, Higher-order thinking skills	N-RN.2, N-Q.1, F-IF.4-9, F-BF.3, A-CED.1, A-CED.2, A-CED.4, A-SSE.3, A-SSE.4, A-APR.3, A-APR.4, A-REI.1-4, F-TF.2, CRP1,2,4,6,7,8,11	*Formative Assessment:* Homework Questioning IXL Classwork Group and Cooperative Work *Summative Assessment* • Test on the Unit Circle, Trigonometric Functions, Algebra Skills • Quiz on finding limits graphically, numerically, and algebraically • Test on evaluating all types of limits including one sided and infinite
Applications of Differentiation ● Extrema on an Interval ● Rolle's Theorem ● Mean Value Theorem ● Increasing and Decreasing Functions and the First Derivative Test ● Concavity and the Second Derivative Test ● Limits at Infinity ● Curve Sketching ● Optimization Problems	20 weeks	*For Support:* Guided Notes, Use of IXL, Use of a Calculator, Modified Assessments Rephrase questions, directions, and explanations, Testing accommodations, *For Enhancement:* Use of IXL, Provide extension activities,	A-CED.1, A-CED.2, A-CED.4, A-SSE.3, A-SSE.4, A-APR.3, A-APR.4, A-REI.1-4, F-TF.2, 8.2.12.A.2 8.2.12.C.3 8.2.12.C.4	*Formative Assessment:* Homework Questioning IXL Classwork Group and Cooperative Work *Summative Assessment* • Quiz on Basic Differentiation • Test on Rolle's Theorem, Mean Value Theorem, and the First and Second Derivative Tests

Unit Topic	**Time Allocated**	**Differentiating Instruction for Students with Disabilities, Students at Risk, English Language Learners, & Gifted & Talented Students**	**Standards**	**Assessments**
• Differentials • Propagated error • Business & Economics		Adjusting the pace of lessons, Higher-order thinking skills, Real-world problems and scenarios	8.2.12.D.4 9.1.12.E.5 CRP1,2,4,6,7,8,11	• Quiz on Sketching a Graph using derivatives • Test on Graph Sketching and Optimization problems • Quiz on differentials and propagated error • Test on differentials, propagated error, and Business and Economics optimization problems
Integration • Antiderivatives and Indefinite Integration • Riemann Sums and Definite Integration • Area Under the Curve • Area Between Curves • The Fundamental Theorem of Calculus • Integration by Substitution • Integration by Parts • Volume by Slicing	15 weeks	*For Support:* Guided Notes, Use of IXL, Use of a Calculator, Modified Assessments Rephrase questions, directions, and explanations, Testing accommodations, *For Enhancement:* Use of IXL, Provide extension activities, Adjusting the pace of lessons, Higher-order thinking skills, Real-world problems and scenarios	F-IF.4-9, F-BF.3, A-CED.1, A-CED.2, A-CED.4, A-SSE.3, A-SSE.4, A-APR.3, A-APR.4, A-REI.1-4, F-TF.2, G-SRT.8, G-GMD.3-4, G-MG.1, G-MG.2 CRP1,2,4,6,7,8,11	*Formative Assessment:* Homework Questioning IXL Classwork Group and Cooperative Work *Summative Assessment:* • Quiz on Antiderivatives • Quiz on Riemann Sums • Quiz on Areas • Test on Fundamental Theorem of Calculus • Quiz on Integration by Parts • Test on Integration
Final Project • Career Research • Practical Applications	1 Week	*For Support:* Use of a Calculator, Modified Assessments Rephrase questions, directions, and explanations, *For Enhancement:*	A-SSE.3, A-SSE.4, A-APR.3, A-APR.4, A-REI.1-4, F-TF.2,	*Formative Assessment:* Questioning Classwork Group and Cooperative Work *Summative Assessment:*

Unit Topic	Time Allocated	Differentiating Instruction for Students with Disabilities, Students at Risk, English Language Learners, & Gifted & Talented Students	Standards	Assessments
		Provide extension activities, Higher-order thinking skills, Real-world problems and scenarios	F-IF.4-9, F-BF.3, 8.1.12.A.1 9.1.12.A.4 9.2.12.C.1 CRP1,2,4,6,7,8,11	• Written paper on what the career entails and how one achieves the position • Presentation teaching what calculus topics are used in a given career and how specifically they are used.