

RUTHERFORD HIGH SCHOOL  
Rutherford, New Jersey

COURSE - AP CALCULUS

**Statement of Purpose/Philosophy:** This Advanced Placement (AP) course in calculus consists of a full high school academic year that is comparable to calculus courses in colleges and universities. Upon completing AP Precalculus as juniors, the students who take this course will continue their study of differential and integral calculus and be prepared to take the College Entrance Examination Board's Advanced Placement Calculus Examination (AB version) in the spring of their senior year. It is expected that most students will seek college credit, college placement, or both, from institutions of higher learning.

This course is concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. It provides a multirepresentational approach to calculus, with concepts, results, and problems being expressed geometrically, numerically, analytically, and verbally. The connections among these methods of representation are also emphasized. (15)

Students will also continue to study the interdisciplinary aspects of mathematics, art, music, philosophy and artificial intelligence by reading and discussing selections from the third Douglas Hofstadter work, Metamagical Themas. (3)

**General Student Objectives:** ( 1- 5, 10,11, 15 -18)

The student will be able to:

- A. Work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal.
- B. Understand the meaning of the derivative in terms of a rate of change and local linear approximation.
- C. Understand the meaning of the definite integral both as the limit of Riemann sums and as the net accumulation of a rate of change.
- D. Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- E. Use derivatives to solve a variety of problems.
- F. Use integrals to solve a variety of problems.
- G. Communicate mathematics both orally and in well-written sentences and explain solutions to problems.
- H. Model a written description of a physical situation with a function, a differential equation, or an integral.
- I. Use technology to help solve problems, experiment, interpret results, and verify conclusions.
- J. Determine the reasonableness of solutions.
- K. Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.
- L. Become more familiar with self-referential sentences.
- M. Relate topics from Godel, Escher, Bach and The Mind's I to the concepts from selected readings in Metamagical Themas.

**Course Outline:**

- I. The Derivative as a Function
  - A. Corresponding characteristics of graphs of  $f$  and  $f'$
  - B. Relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$
  - C. The Mean Value Theorem and its geometric consequences
  - D. The interpretation of Rolle's Theorem

- E. Equations involving first and second derivatives
- II. Second Derivatives
  - A. Relationship between concavity and the graphs of  $f$ ,  $f'$ , and  $f''$
  - B. Points of inflection
- III. Applications of derivatives
  - A. Curve analysis
  - B. Optimization, absolute and relative extrema
  - C. Related rates
  - D. Use of implicit differentiation to find the derivative of an inverse function.
  - E. Derivative as a rate of change in application areas, including velocity and acceleration.
- IV. Integration
  - A. Riemann sums using left, right and midpoint evaluation points
  - B. The definite Integral as a limit of Riemann sums over equal subdivisions
  - C. The definite integral as the change of quantity over the interval
  - D. The basic properties of definite integrals
  - E. Applications of definite integrals
    - 1. area of a region
      - a. Estimates of area by Riemann sum and Trapezoidal Rule
    - 2. distance traveled by a particle
    - 3. volume of a solid of revolution and by cross-section
    - 4. average value of a function
- V. Fundamental Theorem of Calculus
  - A. Use of the Fundamental Theorem to evaluate definite integrals
  - B. Use of the Fundamental theorem to represent a particular antiderivative and its analytical and graphical analysis
- VI. Techniques of Antidifferentiation
  - A. Use of initial conditions
  - B. Powers of odd and even trigonometric functions
  - C. Substitution methods
  - D. Integrals of  $a^2 - u^2$
  - E. Integrals of  $a^2 + u^2$
  - F. Integrals of  $u^2 - a^2$
  - G. Integration by parts
- VII. Applications of Integration
  - A. Solving separable differential equations and using them in modeling real world phenomena, especially, for logarithmic functions and exponential growth.
  - B. Motion equations
- VIII. Reading and Discussion of topics from Metamagical Themas
  - A. Contrasting the purpose, structure and themes among The Mind's I, Godel, Escher, Bach and Metamagical Themas.
  - B. The concept of self-reference and its relation to Artificial Intelligence
  - C. Quines, memes, viruses, and other self-replicating structures.
  - D. Self-modifying structures
  - E. Implications for the "Prisoner's Dilemma" in today's society.

**Evaluation:** (17,18)

Students will take tests and quizzes and provide solutions to problems in class and for homework. They will engage in laboratory activities which afford them the opportunity to discover calculus concepts through hands-on work. The students will work cooperatively with classmates on projects involving the solution of problems, oral explanations, calculator labs and computer labs. Spiral review assignments will ensure continued practice of all important concepts in mathematics learned throughout the student's high school career. Additionally, book reports (both written and oral versions) will be presented as a result of summer enrichment reading. Participation in class discussions is required and expected.

**Activities:**

Students will:

- A. Complete a study of the appropriate chapters from Calculus - Fifth Edition
- B. Participate in laboratory work involving the discovery of calculus concepts
- C. Prepare problem solutions and present them orally to the class
- D. Prepare a written report based on the summer reading of a book from the approved Math Department reading list at the AP level.
- E. Prepare an oral report based on the summer reading of a book from the approved Math Department reading list at the AP level and present it to the class.
- F. Take an essay test based on the class reports of the summer readings.
- G. Visit various calculus web sites and use their materials on line.
- H. Prepare solutions to previous and practice AP exam problems.
- I. Successfully complete in-class tests and quizzes.
- J. Successfully complete spiral review assignments.
- K. Participate in graphing calculator/ computer laboratory activities.
- L. Read selections from Metamagical Themas by Douglas Hofstadter and participate in discussions, complete writing assignments, and take an essay test based on the readings.

**Bibliography:**

Text: Larson/Hostetler Calculus-Fifth Edition D.C. Heath and Company, 1994.

- Broadwin Solutions A.P. Calculus Problems Pt. II AB and BC 1996  
Mathematical Olympiads for Elementary and Middle Schools, 1996.
- Hofstadter Metamagical Themas Basic Books, Inc., 1985.
- Lederman Multiple Choice and Free-Response Questions in Preparation for the AP Calculus (AB) Examination (Seventh Edition)  
D&S Marketing Systems, 1999

- References: Ayres Schaum's Outline Series, Theory and Problems of Differential and Integral Calculus, 2nd edition McGraw-Hill Book Company, 1964.
- Forester Calculus, Concepts and Applications. Key Curriculum Press, 1998

**Technology:**

Mac Bestgrapher

Algebra Xpresser

TI -89Graphing Calculator

Calculus Web sites: 1) <http://www.collegeboard.org/ap/calculus/>

2) <http://www.seresc.k12.nh.us/www/alvirne.html>

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*Note: References to the New Jersey Core Curriculum Content Standards appear as a numeral in parentheses.*